Heat Exchanger Design

The VIA EPIAm computer board, according to its published specifications, is designed to run without a fan. The heat sinks on each of the chips are, under normal circumstances, sufficient to dissipate all of the heat generated simply through free convection heat transfer. Unfortunately, our system will not be operating under normal circumstances.

The enclosure designed in the previous section is extremely compact, and the total volume of the structure has been kept as small as possible. This was necessary in order to provide a maximum level of clearance for shock mount travel without interference. However, the result of this design constraint is a very small amount of air within the closed system available for free convection.

Air is a very limited cooling fluid. It can only absorb 1.0057 $\frac{kJ}{kg \cdot {}^{\circ}C}$, and its density is only $1.1774 \frac{kg}{m^3}$. This means that, for a given volume and pressure, air will have a very low capacity to absorb heat from its surroundings.

Because of the extreme restrictions on airflow within the enclosure, the electronics will probably not be able to dissipate enough heat through free convection alone. Some manner of forced convection must be provided, or else the electronics will get too hot, and we risk unstable behavior.

Because of the size and power of the robot that these electronics will be controlling, unstable behavior is absolutely unacceptable. We must be sure that the electronics are not allowed to exceed a stable temperature, but we also must ensure that no moisture is allowed to enter the compartment, as this would also compromise the reliability of the system.

In order to accomplish this task, we have designed a very simple heat exchanger. It will be constructed of aluminum tubing, and, in order to maximize surface area for heat dissipation, will run along the perimeter of the robot. This exchanger will be fed by hot air that will be pumped out of the electronics compartment. After passing through the tubing, the cooled air will be routed back into the enclosure.

This solution, it is hoped, will be able to dissipate the heat generated without ever opening the system to the environment, so we can be sure that the electronics will remain dry and contaminant-free. However, we must now determine a configuration that will perform as desired, and for that, we need to know several things.

First of all, we must know just how much heat we need to dissipate. With a little research, we found that the EPIAm board we are using can consume up to around $15\,W$, a hard disk such as ours consumes $10 - 15\,W$, and a normal USB device like a webcam will draw less than $5\,W$.

Also, the blowers under consideration to pump the air can be expected to generate roughly $3\,W$ of power, some of which must be dissipated as heat. Based on these estimations, we will design a heat exchanger capable of dissipating $35\,W$ of heat.

We also must know what temperature will yield unstable behavior. Based on published component documentation as well as personal experience, we are going to shoot for a processor temperature of $60°C$. This is a fairly safe temperature. A case study of workstation overheating problems did not demonstrate random boot-failures and serious destabilization symptoms until the processor reached a temperature of $100°C$. Because of this, we can be confident that at $60°C$, the system will run reliably.

This is, however, a chip temperature. If the air is allowed to reach $60°C$, the processor will quite likely be much hotter than this. We played around with the numbers and some preliminary heat transfer equations for a while before things began to work out.

We adjusted the temperature increase allowed to occur, which impacted the flow rate necessary to cool the system, which effected the flow regime, which effected the necessary heat exchanger geometry, which in turn effected the temperature increase. Simply put, there were far more variables than equations, and we played around with the system's configuration for quite a while before properly constraining it.

Mass Flow Rate

In an early iteration of this process, we were rather optimistic, shooting for a $12°C$ increase in air temperature between inlet and exit. With this, we can easily calculate the necessary mass flow rate from the following:

$$q = 35\,W = \dot{m}C_v(\Delta T) = \dot{m}\,\frac{kg}{s} \cdot (1005.7\,\frac{J}{kg \cdot^\circ C}) \cdot (12°C)$$
$$\dot{m} = 0.0029\,\frac{kg}{s}$$

This calculation assumes a worst case scenario. That is, no heat is lost from the enclosure itself. This makes the problem easy to approach in two steps, and also serves to provide a safety factor. We will first determine the volumetric flow rate necessary to remove the heat from the sealed, adiabatic enclosure. Then, given that flow rate, we will determine the heat convected away from the heat exchanger, and the equilibrium temperature.

Now, based on the published properties of air at atmospheric pressure, we can calculate the volumetric flow rate as shown bellow:

$$\dot{V} = 0.0029\,\frac{kg}{s} \cdot 0.8493\,\frac{m^3}{kg} = 0.0024632\,\frac{m^3}{s} = 5.22\,cfm$$

This flow rate must move through the heat exchanger tubing itself. This was another very optimistic design, as we will see here. The design called for two flexible tubes leading from the enclosure to the heat exchanger, at which point, six aluminum tubes would provide the convection surface. The flexible tubing's total cross sectional area was $0.3927\,in^2$, while the aluminum tubing's area was only $0.2771\,in^2$, making the rigid portion of the design the most restrictive.

Calculating the average linear velocity through the tubing with this configuration is done as follows:

$$5.22\,\frac{ft^3}{min} \cdot \frac{min}{60\,s} \cdot \frac{(12\,in)^3}{ft^3} = 150.336\,\frac{in^3}{s}$$
$$\bar{V} = 150.336\,\frac{in^3}{s} \cdot \frac{1}{0.2771\,in^2} = 542.533\,\frac{in}{s} \approx 31\,mph$$

This is much, much too fast. It is desirable to maintain a laminar flow throughout the system, and this flow is much to fast and too restricted. It was clear at this point that we needed a larger cross sectional area, and a wider acceptable range of temperatures.

Because we had so much elbow room to start with, the latter was no problem. With a $20°C$ increase within the enclosure, the system can still be expected to stay cool enough to keep the processor below $60°C$.

Also, there is some more room in the design for a larger piping array. By rerouting some of the tubes and adjusting some of the mounts, we have been able to greatly increase the cross sectional area of the rigid tubing, from $0.2771\,in^2$ to $0.7261\,in^2$.

The previous process is repeated here with the new parameters dictated by the configuration update.

$$q = 35\,W = \dot{m}C_v(\Delta T) = \dot{m}\,\frac{kg}{s}\cdot(1005.7\,\frac{J}{kg\cdot^\circ C})\cdot(20^\circ C)$$
$$\dot{m} = 0.00174\,\frac{kg}{s}$$
$$\dot{V} = 0.00174\,\frac{kg}{s}\cdot 0.8493\,\frac{m^3}{kg} = 0.001478\,\frac{m^3}{s} = 3.1315\,cfm$$
$$3.1315\,\frac{ft^3}{min}\cdot\frac{min}{60\,s}\cdot\frac{(12\,in)^3}{ft^3} = 90.18795\,\frac{in^3}{s}$$
$$\bar{V}_5 = 150.336\,\frac{in^3}{s}\cdot\frac{1}{0.7261\,in^2} = 124.21\,\frac{in}{s}\approx 7\,mph$$
$$\bar{V}_3 = 150.336\,\frac{in^3}{s}\cdot\frac{1}{0.9204\,in^2} = 97.99\,\frac{in}{s}\approx 5.5\,mph$$

The system, as designed for the above calculations, consists of three flexible tubes, with inside diameters of $0.625\,in$, leading from the enclosure to the heat exchanger, and five rigid aluminum tubes, with inside diameters of $0.43\,in$, making up the heat exchanger itself. The subscripts "3" and "5" refer to the three tube portion and the five tube portion respectively.

Flow Regime

The speeds resulting from the above work are in a much more acceptable range than those of the previous design, and are very promising. To make sure, we simply need to calculate the flow regime in each portion of the system. The Reynold's numbers are calculated below:

$$Re_5 = \frac{\rho\bar{V}D}{\mu} = \frac{(1.1774\,\frac{kg}{m^3})(3.155\,\frac{m}{s})(0.0109\,m)}{(1.8\cdot 10^{-5}\,\frac{N\cdot s}{m})} = 2253.95$$
$$Re_3 = \frac{\rho\bar{V}D}{\mu} = \frac{(1.1774\,\frac{kg}{m^3})(2.489\,\frac{m}{s})(0.016\,m)}{(1.8\cdot 10^{-5}\,\frac{N\cdot s}{m})} = 2584.53$$
$$2300 \leq Re_{crit} \leq 5000 \Rightarrow \text{The flow may be treated as laminar}$$

As you can see above, the flow in the aluminum, five-tube portion of the exchanger remains fully laminar ($Re \leq 2300$). The flow through the flexible tubing, however, begins to become transitional. It remains in the very lower regions of transitional flow behavior, and, because it is in a non-critical portion of the heat exchanger, it can be treated as laminar without significant loss in accuracy.

Head Loss

Now that we know the flow regime throughout the system, we can calculate the head loss at each location in the heat exchanger. This is necessary in order to determine the total head loss, which will be used to select our pump. We must be able to find an air pump capable of delivering enough pressure to overcome the head loss in the system while still delivering the desired $3.1315\, cfm$ of volumetric flow rate.

At the pump exit, an adapter plate will be made to direct the flow into the three flexible lengths of tubing. This plate will have filleted inside corners with radius of at least $0.015\, in$, meaning that the head loss at this juncture is given by the following:

$$H_l = 0.15 \cdot \frac{\bar{V}^2}{2g} = 0.15 \cdot \frac{(97.99\, \frac{in}{s})^2}{2 \cdot (386.4\, \frac{in}{s^2})} = 1.86375\, in_{air}$$

After this junction, the flow will enter the first section of flexible tubing. Based on its geometry, this has been modeled as a $90°$ pipe bend with a radius of $1.5\, in$, giving us the following:

$$H_l = f \frac{L_e}{D} \frac{\bar{V}^2}{2g} = 0.025 \cdot \left(12\, \frac{in}{in}\right) \cdot \frac{(97.99\, \frac{in}{s})^2}{2 \cdot (386.4\, \frac{in}{s^2})} = 3.7275\, in_{air}$$

After the flexible tubing, the flow will pass through an adapter, where the three flexible tubes are split into the five rigid tubes. This will have the same kind of internal geometry as the pump exit plate, but with a higher average velocity. The head loss here is given by:

$$H_l = 0.15 \cdot \frac{\bar{V}^2}{2g} = 0.15 \cdot \frac{(124.21\, \frac{in}{s})^2}{2 \cdot (386.4\, \frac{in}{s^2})} = 2.9946\, in_{air}$$

Following this juncture, the flow enters the aluminum tubing. This is by far the most complicated and critical portion of the process. This is where the majority of head loss will be found, and where the most bends and corners are found. The first step in calculating the head loss across this system is determining the equivalent length of the pipe. This is the length of a pipe with no bends or elbows that has the same amount of head loss. From this, it is very simple to calculate the head loss.

$$L_e = 2[37.1537 + 14 \cdot D + 2(4 \cdot D) + 12 \cdot D] = 103.5\, in$$

$$H_l = f \frac{L_e}{D} \frac{\bar{V}^2}{2g} = 0.0284 \frac{(103.5\, in)}{(0.43\, in)} \frac{(124.21\, \frac{in}{s})^2}{2(386.4\, \frac{in}{s^2})} = 136.5032\, in_{air}$$

Following the rigid tubing, the flow enters another juncture, another length of flexible tubing, and finally, enters the computer enclosure. The head loss at each of these stages are set up the same as before, except that this length of flexible tubing is modeled as a 180° bend, due to its geometry.

$$H_l = 0.15 \cdot \frac{\bar{V}^2}{2g} = 0.15 \cdot \frac{(97.99\,\frac{in}{s})^2}{2 \cdot (386.4\,\frac{in}{s^2})} = 1.86375\,in_{air}$$
$$H_l = f\frac{L_e}{D}\frac{\bar{V}^2}{2g} = 0.025 \cdot \left(24\,\frac{in}{in}\right) \cdot \frac{(97.99\,\frac{in}{s})^2}{2 \cdot (386.4\,\frac{in}{s^2})} = 7.455\,in_{air}$$

The total head loss in the system is the sum of the head losses in each of its parts. The result is given in inches of the working fluid (air, in this case), but can easily be converted to inches of water using the ratio of specific gravities.

$$H_{l,total} = \Sigma H_l = 154.4078\,in_{air} \cdot \frac{1.1774\,in_{H_2O}}{1000\,in_{air}} = 0.1818\,in_{H_2O}$$

This is a very encouraging result. This means that, in order to cool the electronics with this heat exchanger, we simply need a pump that can provide $0.1818\,in_{H_2O}$ of pressure while delivering $3.1315\,cfm$ of volumetric flow rate. This can be achieved through the use of a computer case blower, which also has an excellent form factor for our purposes, and can even draw directly from our power supply.

Free Convection Heat Transfer

There is simply one more concern, but it is very significant. We need to know if the aluminum tubing can dissipate $35\,W$ of heat in a single pass, and what temperature the system will stabilize at. This is not a simple calculation.

After some significant number crunching and simplification, we have generated the following 13 relationships, expressions, and equations that will be useful in solving this problem:

$$q = \bar{h}A(\Delta T) \qquad \bar{h} = \bar{Nu}\frac{k}{x} = 2.066 \cdot \bar{Nu}$$

$$\bar{Nu} = 0.85(GrPr)^{0.188} \qquad T_o = T_i + 20$$

$$Gr = \frac{\rho^3 g\beta(\Delta T)x^3}{\mu^2} \qquad \Delta T = T_i - 13$$

$$Pr_{air} = 0.708 \qquad T_f = \frac{T_i}{2} + 15$$

$$\rho_{air} = 1.1774\,\frac{kg}{m^3} \qquad \beta = \frac{1}{T_f}$$

$$k_{air} = 0.02624\,\frac{W}{m \cdot^\circ C} \qquad \mu_{air} = 1.8462 \cdot 10^{-5}\,\frac{kg}{m \cdot s}$$

$$A = 5\pi dL = \frac{5\pi}{2}(8.3468)\,in^2 = 655.556\,in^2$$

This gives us the interrelationships that exist between all of the major factors under consideration. The first line contains the general form of convection heat transfer and the definition of the average Nusselt number ($\bar{Nu}$). The second line contains the main, driving equation, which gives the average Nusselt number in terms of the Grashof number (Gr) and the Prandtl number (Pr). Solving this equation will give us almost all of the information we need.

Also included are a number of different temperatures of interest, including the outlet temperature (T_o), inlet temperature (T_i), difference between the local temperature and ambient (ΔT), and the film temperature (T_f). Also, the definition of the Grashof number is given, as well as some properties of air and the total area of the heat exchanger.

All of these equations can be easily manipulated and combined together in order to isolate a single variable. Three pages of algebra later, we have an expression of T_{in} only:

$$T_{in_{(n+1)}} = 47.113849\Big(115726.813 - \frac{68163092.87}{T_{in_{(n)}} + 576}\Big)^{-0.188} + 13$$

This is a very straightforward numerical analysis problem, and it converges in thirteen iterations within $1 \cdot 10^{-5}\,^\circ C$ to:

$$T_{in} = 24.13642^\circ C \approx 75^\circ F$$

This, based on the mass flow rate, directly gives us:

$$T_{out} = 44.13642°C \approx 112°F$$

This means that the temperature inside the enclosure should never rise above $44.2°C$, which is well within acceptable limits. This will provide more than $15°C$ temperature difference between the cooling air and the processor before we reach our critical value, which should be plenty of elbow room.

Conclusions

So, with the above calculations, we have shown that every aspect of this heat exchanger should meet our needs. It accommodates the shock mountings of our electronics with flexible tubing, it does not compromise the water-tight seal on the enclosure, it dissipates all of the heat generated without reaching unacceptable temperatures, and it has low enough head loss and flow rate to be supplied by a simple, unmodified, readily-available commercial blower.